# Voidage Variation in Packed Beds at Small Column to Particle Diameter Ratio

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*Randomly packed beds of equal spheres exhibit damped oscillatory voidage variation in the near wall region. Models reported in literature were found to be lacking in their ability to relate this variation to average bed porosity. The changing mean of the voidage oscillations and deviations from sinusoidal behavior were not described well either. Experimental data and a new radial bed porosity model are presented. Improved radial bed voidage and average bed voidage predictions were demonstrated. It has further been shown that multiple stable packing configurations exist within the same packing mode, which complicates modeling at a small column to particle diameter ratio.*

## Introduction

The voidage variation of packed beds in the near wall region has been studied extensively because of its influence on pressure drop, bed permeability, fluid hold-up, linear velocity, and residence time distribution. At small column to particle diameter ratios, typically less than ten, this effect becomes more prominent. Since it is the aim of many laboratories to conduct test work at the smallest practical scale, it is important to be able to accurately quantify the voidage variations at small column to particle diameter ratios.

Various mathematical models to describe radial porosity variation can be found in literature, and they generally seem to be in good agreement with experimental data. It was therefore surprising to find a lack of consensus in literature regarding the influence of column to particle diameter ratio on bed permeability (Cohen and Metzner, 1981; Nield, 1983, 1985; Cohen, 1985; Chu and Ng, 1989, 1990; Tsotsas and Schlünder, 1990; Winterberg and Tsotsas, 2000). Since permeability in the wall region is governed by two competing phenomena, namely, porosity and viscous wall friction, it was decided to re-evaluate the description of bed porosity as a potential source of uncertainty.

Scrutiny of the experimental radial porosity variation data in literature indicated a possible bias in the models. Although the goodness of fit between the model predictions and radial porosity variation data was generally good, integration of the radial porosity functions yielded widely differing average bed porosity values at small column to particle diameter ratios. In an attempt to clarify these issues, literature data is revisited and some new experimental data and correlations are presented.

## Literature

### *Experimental investigations of radial porosity variation*

The voidage variation of packed beds in the near wall region has been experimentally investigated by a number of researchers. Although the experimental work was approached in many different ways, the results are in general agreement:

Roblee et al. (1958) were some of the first researchers to measure the radial porosity variation in the near wall region experimentally. This was done by packing cardboard cylinders with cork spheres and filling the void space with molten wax. After solidification, sections were cut out and the wax fraction in each section was determined. They found that the bed porosity showed damped oscillatory behavior in the near wall region, reaching a constant value about 4 to 5 particle diameters from the wall. The maxima and minima of the voidage oscillations were observed at integral multiples of the particle radius. Considerable experimental effort went into their work, and the results were reported with a 95% confidence interval of 0.025 void fraction units.

Benenati and Brosilow (1962) filled a container with uniformly sized lead shot and then filled the void space with liquid epoxy resin. After curing of the resin, the solid cylinder was machined and the weight loss noted. Essentially, the same behavior as noted by Roblee et al. (1958), was found. The voidage varied in an oscillatory fashion with the amplitude decreasing with increasing distance from the wall. No oscillations were noted at a distance further than 4.5 particle dia. from the wall.

Scott (1962) determined the position of steel balls fixed in a wax matrix to calculate the radial porosity distribution. Although the uncertainty in the radial position used to calculate the void fraction was 20% of the particle diameter, the data showed damped oscillatory behavior that was extinguished about 4 particle dia. from the wall.

Thadani and Peebles (1966) made use of radiography to study the voidage variation in a packed bed. They found a definite cycling tendency in voidage that was damped towards the center of the bed, although the voidage variation in the region up to 2.5 particle dia. from the wall was less pronounced than that reported by previous researchers. Unfortunately, the large variance ($s^2$) of their experimental data detracted from its usefulness.

Ridgway and Tarbuck (1966, 1968) followed yet another approach whereby a cylinder with balls was centrifuged and the level increase with liquid addition was noted. The results thus obtained displayed a damped oscillation similar to that noted by Roblee et al. (1958) and Benenati and Brosilow (1962), although steady values were only obtained between 5 and 6 particle diameters from the wall. More recently, Giese et al. (1998) also made use of this approach.

Pillai (1977) used a transparent two-dimensional (2-D) bed that was filled with wooden discs and noted the position of each disc. Although a damped oscillatory behavior was observed, his results cannot be directly compared with those of the other researchers, since random packed beds are not ordered in the radial-axial-plane.

Staněk and Eckert (1979) measured the change in liquid level of a large diameter cylinder packed with glass spheres on injecting discrete quantities of ethanol. Their objective was to measure porosity variation using the bottom wall of the cylinder as a reference, but noted that the porosity variation induced by the cylinder wall could also influence the quality of their data. Damped oscillatory behavior was noted up to 6 particle diameters from the wall. The oscillations resembled a series of parabola joined in such a way that sharp high voidage edges were formed between each parabola, smoothing only after 3 to 4 particle dia. from the wall. This shape was characteristic of most of their porosity profiles.

Goodling et al. (1983) essentially used the experimental method of Benenati and Brosilow (1962), but used polystyrene balls in an iron powder and epoxy resin mixture. Their results displayed damped oscillatory behavior, but, in many of their experiments, the form of the oscillations resembled a repetition of parabola, rather than being sinusoidal. Steady voidage values were obtained within 5 particle dia. from the wall.

Küfner and Hofmann (1990) filled the space between the particles in a packed bed with a resin and, after curing of the resin, the tube was cut into layers. The cuts were polished and photographed. Photographs of the cuts were analyzed with image analysis software to determine the radial porosity distribution. Damped oscillatory behavior was observed.

Mueller (1992) used precision instruments to insert stainless steel microspheres into plexiglas balls. Radiography was used to determine the position of the balls in a randomly packed bed, and a lot of care was taken to eliminate potential distorting influences. The oscillatory behavior resembled a repetition of parabola, smoothing to a more sinusoidal form about 3 particle diameters from the wall.

### *Experimental investigations of average bed porosity variation with column to particle diameter ratio*

Although the influence of column to particle diameter ratio on the voidage of packed beds can be calculated from the radial porosity variation data, some researchers have also investigated this relationship experimentally. Such determinations are less arduous and the methods employed were similar in nature.

Carman (1937) used a lead shot that was shaken down to its closest possible packing in glass tubes of different diameter. The parabola shaped voidage variation predicted from geometrical calculations was obtained in the column to particle diameter ratio region of $1 \le D/d \le 2$. An exponential decline was observed in the region $2 < D/d < 10$, and constant voidage values were obtained at larger column to particle diameter ratios.

Leva and Grummer (1947) filled standard steel pipes with a known quantity of smooth glass, steel, or porcelain spheres. By noting the height, the voidage of the packing could be determined. This was done for "loose" and "dense" packing, with the "dense" packing having a voidage on average 4% (4.2% $\pm$ 2.4) less than the "loose" packing, but the same general trend. The average deviation of repeat runs was 1%. They found a linear relationship between the inverse ratio of column to particle diameter and the bed voidage, quite similar to an exponential decay function.

### *Modeling of bed porosity variations*

Various attempts at modeling these voidage variations have been presented in literature. Most of the more recent models describe both the oscillatory nature and damping of the voidage variations. However, some simplified models, like the model of Vortmeyer and Schuster (1983), describe the voidage increase in the near wall region in terms of an exponential-type function only.

The models suggested by Martin (1978), Cohen and Metzner (1981), and Küfner and Hofmann (1990) are similar in the sense that they all contain a cosine term to describe the oscillations and an exponential term to describe the dampening. The influence of the column to particle diameter ratio on the period of oscillation was recognized by Martin (1978) and Cohen and Metzner (1981), although only the former included it in his model.

Mueller (1991, 1992) modeled the oscillations with a zero-order Bessel function of the first kind and described the dampening with an exponential term. The effect of column to particle diameter ratio on the period of the oscillations was taken into account.

Govindarao and Froment (1986), Govindarao and Ramrao (1988), and Kubie (1988) described the voidage variation in terms of the fraction of spheres with centers laying in cylindrical concentric layers of the bed. The description becomes less accurate as distance from the wall increases.

More recently, computer models, like that of Nandakumar et al. (1999), have been used to simulate randomly packed beds to investigate voidage variation.

## Experimental

*Apparatus:* All experiments were conducted with nonporous glass beads of 0.003 m diameter. These were characterized in terms of their size variation and density. Glass columns, 0.6 m in length, and with an internal dia. ranging from 0.0049 m to 0.0575 m, were used.

*Method:* The container of glass beads was sampled and a number of beads were weighed individually on an electronic analytical balance (Sartorius BP211D, readability $1\times10^{-8}$ kg) to determine their size variation. The density of the glass beads was determined by distilled water (conductivity of $2\times10^{-4}$ S/m) displacement in a 10 ml A grade volumetric flask. The volume of the volumetric flask was verified by blank measurements with distilled water only. Temperature corrections were done, and the temperature of the water was measured with a mercury thermometer (readability of 0.1 K, uncalibrated).

The bed voidage was determined by measuring the mass of glass beads contained in a glass column of known volume. The mass of the glass beads was related back to their volume through their density. The void volume could be calculated from the difference between the column volume and that occupied by the glass beads. The glass beads were poured into the column with some tapping of the column wall. The glass beads contained by the column were weighed on an electronic balance (Sartorius MP8-1, readability $1\times10^{-4}$ kg).

## Results

The experimental values obtained for the size variation and density of the glass beads are presented in Table 1. The average diameter of the glass beads ($2.98\times10^{-3}\pm2\times10^{-5}$m) can be calculated from these data. An exact measurement of sphericity was not done, but visual inspection indicated no irregular shapes and a sample of glass beads checked by measurement with a Vernier calliper (readability of $5\times10^{-5}$ m) indicated no irregularities outside of the readability of the instrument.

The experimental bed voidage values for various column to particle diameter ratios are presented in Table 2. It should be noted that the sample standard deviation of the porosity measurement refers to the variation in the calculated values based on the average properties of the glass beads used. It is, therefore, not a true representation of the confidence in the absolute value of the porosity, which should be somewhat larger.

## Discussion

Most of the literature reviewed, as well as the present experimental data presented, is focused on packed beds of spheres. Although some previous researchers, like Roblee et al. (1958), indicated that cylindrical particles exhibit similar behavior, the discussion will be limited to packed beds of equal sized spheres.

**Table 1. Size Variation and Density of Glass Beads**

| | *n* | *x* | *s* |
|---|---|---|---|
| Size variation (kg) | 40 | $3.501\times10^{-5}$ | $8.1\times10^{-7}$ |
| Density (kg/m$^3$) | 10 | 2,520 | 30 |

**Table 2. Bed Voidage at Different Column to Particle Diameter Ratios**

| Col. to Particle Dia. Ratio | Bed Voidage | | |
|---|---|---|---|
| | *n* | *x* | *s* |
| 1.7 | 12 | 0.657 | 0.004 |
| 2.0 | 12 | 0.502 | 0.004 |
| 2.4 | 12 | 0.471 | 0.002 |
| 2.6 | 12 | 0.483 | 0.003 |
| 3.0 | 10* | 0.416 | 0.002 |
| 3.3 | 12 | 0.450 | 0.003 |
| 3.7 | 12 | 0.445 | 0.002 |
| 4.2 | 12 | 0.425 | 0.004 |
| 4.4 | 12 | 0.426 | 0.002 |
| 4.6 | 12 | 0.406 | 0.003 |
| 4.9 | 12 | 0.419 | 0.001 |
| 5.4 | 12 | 0.411 | 0.001 |
| 7.2 | 12 | 0.397 | 0.001 |
| 9.2 | 12 | 0.368 | 0.001 |
| 11.2 | 12 | 0.366 | 0.001 |
| 13.3 | 12 | 0.362 | 0.001 |
| 15.4 | 12 | 0.363 | 0.001 |
| 18.0 | 12 | 0.362 | 0.001 |
| 19.3 | 12 | 0.363 | 0.000 |

*Two values (0.497 and 0.562) were not used for the calculation of the average, but are true values and they have some bearing on the discussion.

### *Packing mode*

Literature values vary for the bed voidage of a randomly packed bed of spheres at large column to particle diameter ratio. This variance is ascribed to the packing mode and has been discussed by authors like Haughey and Beveridge (1969) and McWhirter et al. (1997). Four random packing modes for spheres are distinguished:

(a) Very loose random packing ($\epsilon\approx0.44$): obtained by gradual defluidization of a fluidized bed or by sedimentation.

(b) Loose random packing ($\epsilon\approx0.40-0.41$): obtained by letting spheres roll individually in place, or by dropping the spheres into the container as a loose mass.

(c) Poured random packing ($\epsilon\approx0.375-0.391$): obtained by pouring spheres into a container.

(d) Dense random packing ($\epsilon\approx0.359-0.375$): obtained by vibrating or shaking down the packed bed.

The experimental data for large column to particle diameter ratios reported in Table 2 indicate dense packing ($\epsilon\approx0.362-0.363$).

This has some bearing on the subsequent mathematical treatment of the bed porosity, since its absolute value is a function of the loading methodology used. It also has implications for the comparison of mathematical models for predicting the bed voidage, since packing mode is not an easily quantifiable parameter.

### *Stable packing configurations*

Table 2 indicates apparently anomalous bed voidage values at column to particle diameter ratios of 2.4, 3.0, and 4.6. Although low sample standard deviations were found, in one case two data points were recorded that deviated considerably from the average. Similar behavior was reported by Leva and Grummer et al. (1947) for "loose" and "dense" dense random packing (see Figure 1).

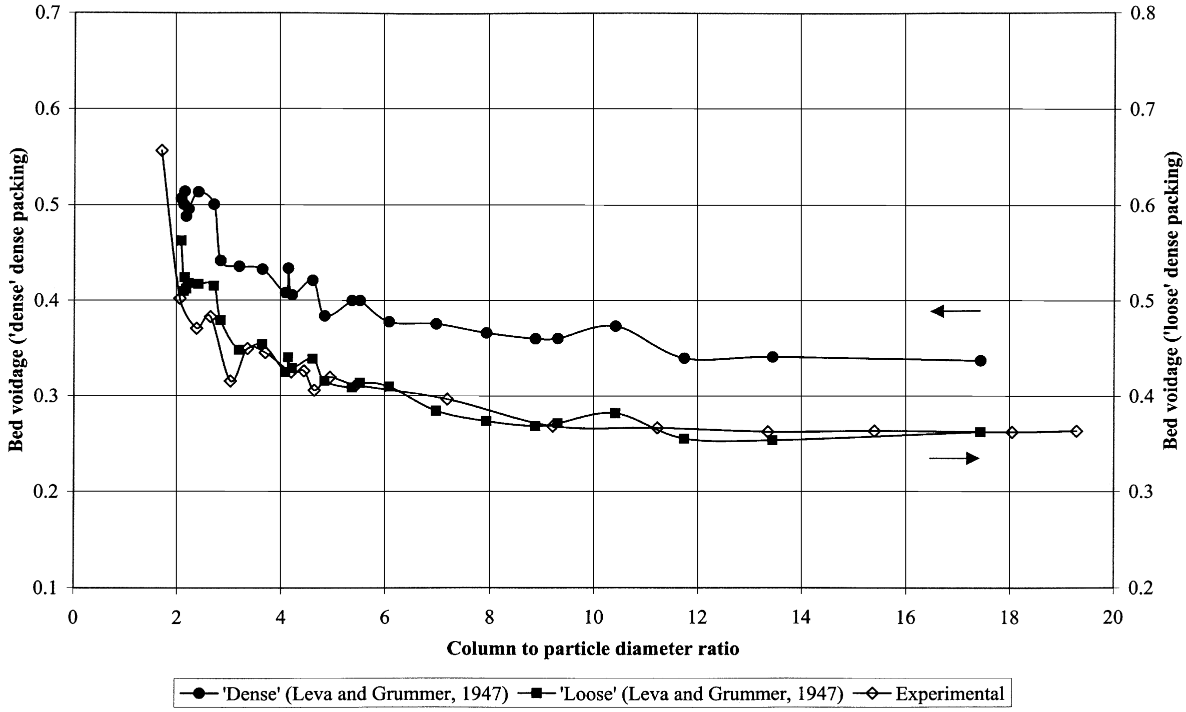


**Figure 1. Average bed porosity as a function of column to particle diameter ratio.**

The variation in bed porosity introduced by the packing mode has already been discussed. It is therefore conceivable that the container wall could also lead to some form of variation. Leva and Grummer (1947) have shown that these variations are superimposed on the effect of packing mode, by presenting data for a "loose" and a "dense" dense random packing.

At small column to particle diameter ratios, where the effect of the wall is more pronounced, different stable packing configurations seem to be possible within a packing mode.

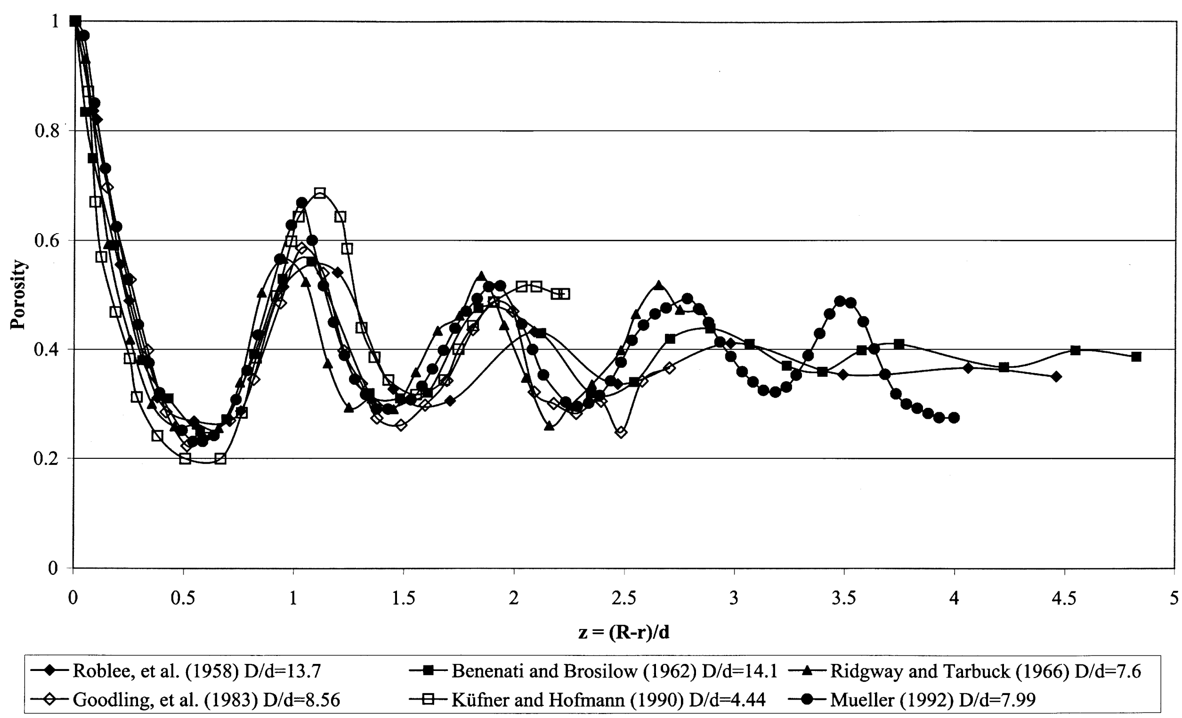


**Figure 2. Radial bed porosity variation.**

Carman (1937) has already hinted that more than one stable packing configuration might be possible in the region $1.866 < D/d < 2$ and, therefore, presented calculations for 1 ($\epsilon = 0.333$) $\leq D/d \leq 1.866$ ($\epsilon = 0.617$) and 2 ($\epsilon = 0.528$) only. He also showed that the bed porosity goes through a maximum at $D/d = 1.667$ ($\epsilon = 0.678$).

At $D/d > 2$, there are regions where more than one stable packing configuration can be found. Calculations similar to those of Carman can be performed at some intermediate values of $D/d$ like 2.154 ($\epsilon = 0.568$), 2.225 ($\epsilon = 0.461$), and 2.414 ($\epsilon = 0.456$). If these values are then compared with the data presented by Leva and Grummer (1947) and those in Table 2, it shows that further porosity maxima and minima are possible. From the same body of experimental data, it has been shown that such behavior is common in the region $2 < D/d < 5$, but that at larger column to particle diameters it cannot be ruled out.

This presents a serious obstacle to the modeling of bed porosity variation at small column to particle diameter ratio. Rapid oscillation of bulk voidage values are possible, and, at best, this can be described as a band of porosity values for a specific packing mode.

### *Radial porosity variation*

Much of the experimental work on near wall porosity variation was aimed at determining the actual voidage variation as a function of distance from the retaining wall. The data obtained by different experimental approaches were generally in agreement on the following (see Figure 2):

(a) The voidage varied in an oscillatory fashion.

(b) The amplitude of the voidage oscillation was increasingly damped with distance from the wall.

(c) The voidage variations were completely damped within 4–6 particle dia. from the wall, although some erratic variation still remained.

(d) The first porosity minimum is at about a half particle diameter from the wall.

(e) The first porosity maximum is approximately one particle diameter from the wall.

Data sets for radial porosity variation look orderly when studied in isolation. Cursory inspection of the data suggests that the oscillations can be adequately described by a function with exponentially damped sinusoidal behavior (which forms the basis for most models reported in literature). However, despite the encouraging goodness of fit reported for these models, the description is an oversimplification.

The first layer of packing in contact with the wall is very structured, but subsequent layers do not necessarily retain this level of order. The packing configuration determines the way in which subsequent layers are packed, and the degree of randomness increases with distance from the wall. The following deviations from exponentially damped sinusoidal behavior have been noted:

(a) The mean of the oscillatory behavior is not a constant value, but rather a decay function.

(b) The form of the oscillations is not always sinusoidal, but can sometimes resemble a series of parabola.

(c) The period of oscillation is not necessarily constant.

Although these deviations do not markedly affect the qualitative description of radial porosity variation, the quantitative description of the radial porosity variation at a distance further than one particle diameter from the wall becomes tenuous. This is of consequence for modeling.

## Modeling

Since the data can be categorized as either average bed porosity data $\epsilon$ or radial bed porosity data $\epsilon(r)$, two different approaches were followed during the model development.

### *Average bed porosity model*

The parameters influencing average bed porosity were identified by Leva and Grummer et al. (1947). They noted that, for regular shaped particles, the average bed porosity was influenced by: packing mode; column to particle diameter ratio; particle shape; particle-size distribution and roughness of the particle surface.

The present work is concerned with equal sized spheres, which limits the generality of the model, but eliminates two of these variables, namely particle shape and particle-size distribution. The roughness of the particle surface influences the ease with which a stable packing configuration can be achieved. It also influences the average bed voidage for a specific packing mode, because particles with a rough surface will have a higher resistance to the slippage needed for denser packing. Handling particle roughness quantitatively as a separate parameter is difficult, and it was, therefore, decided to lump its effect with that of the packing mode. This leaves only two parameters to model: packing mode and column diameter to particle diameter ratio.

The average bed porosity is modeled by an exponential decay function (see Eq. 1). The effect of the packing mode is embodied in the average bed porosity at infinite column diameter ($\epsilon_b$). No attempt was made at providing a correlation to relate $\epsilon_b$ to the packing mode or the particle roughness. The constants were determined by regression of the data by Carman (1937) due to its consistency and spread.

$$\epsilon = \epsilon_b + 0.35 \exp\left(-0.39 \frac{D}{d}\right) \tag{1}$$

The present experimental data, as well as that of Leva and Grummer (1947), contain average bed voidage values for different stable packing configurations. The aim of the model

**Table 3. Average Bed Porosity Predicted by Eq. 1**

| Source of Exp. Data | Relative Estimation Error (%) | | | |
|---|---|---|---|---|
| | $n$ | $x$ | $s$ | Max. |
| Carman (1937) | 7 | 1 | 1 | 2 |
| Leva and Grummer (1947), "loose" | 27 | 4 | 2 | 10 |
| Leva and Grummer (1947), "dense" | 27 | 3 | 3 | 9 |
| Roblee et al. (1958) | 2 | 7 | - | 13 |
| Benenati and Brosilow (1962) | 4 | 4 | 3 | 7 |
| Ridgway and Tarbuck (1966) | 3 | 7 | 3 | 11 |
| Goodling et al. (1983) | 3 | 11 | 3 | 13 |
| Küfner and Hofmann (1990) | 2 | 6 | - | 9 |
| Mueller (1992) | 4 | 13 | 2 | 15 |
| Exp. data, Table 2 | 18 | 2 | 3 | 13 |
| Weighted average | | 4 | 4 | |

was to obtain a good description of the influence of column to particle diameter ratio on the bed voidage, rather than to achieve the least error of fit for all data. Despite this approach, the model describes well the average bed voidage data, as well as area integrated radial bed voidage data (see Table 3).

The model has the following limitations that should be kept in mind when using it:

(a) It is only valid for equal sized spheres.

(b) It has been developed for column to particle diameter ratios larger than 2. The equations of Carman (1937) should be used in the region $1 \leq D/d \leq 2$.

(c) The absolute value of the average bed voidage is sensitive to the choice of $\epsilon_b$, which is a function of packing mode and particle roughness. Incorrect $\epsilon_b$ values will lead to constant over or under prediction of the average bed voidage.

(d) This model does not describe variations due to different stable packing configurations for the same packing mode.

### *Radial bed porosity model*

Three objectives were set for the development of a new radial bed porosity model. The first objective was to address the shortcomings identified in describing the radial porosity variation by an exponentially damped sinusoidal function. The second objective was that the $\epsilon(r)$-model must also be able to describe the average bed porosity well (see Eq. 2). The third objective was to keep the model as simple as possible.

$$\epsilon = \frac{\int_0^R \epsilon(r) \cdot (2\pi r) \cdot dr}{\pi R^2} \tag{2}$$

The exponentially damped sinusoidal form of the model used by Martin (1978), Cohen and Metzner (1981), and Küfner and Hofmann (1990) was retained. In addressing the shortcomings of such a description, the following was found:

(a) The changing mean of the oscillatory behavior could be described by an exponential decay function, without adding too much to the complexity of the model. The constants of the decay function were found to be independent of the description of the oscillatory behavior and could easily be determined. This term was found to be a function of the column to particle diameter ratio only.

(b) Describing the oscillations as a series of parabola can be done by either a repetitive discontinuous function, or its corresponding Fourier series. Not all data displayed this functional form, and, in most cases, the parabolic oscillations tended to smooth out to sinusoidal-like functions further from the wall. It was decided that both approaches would unduly add to the complexity of the model, and the standard sinusoidal description was retained. However, this oversimplification may lead to the average bed voidage being biased towards higher voidage values.

(c) Although the period of oscillation is not constant, no correlating parameter could be found to describe the variation. Martin (1978) and Mueller (1992) correlated it to $D/d$, but this does not hold true for all the data. For modeling purposes, it was decided to take the period of oscillation as constant.

The first layer of particles adjacent to the wall is not only highly ordered, but differs from subsequent layers, because the interstitial space between the wall and the first layer cannot be partially occupied by other particles. It can, therefore, be expected that the form of the radial porosity function for the first half particle diameter from the wall should be different from the rest of the bed. The model takes these differ-

**Table 4. Radial Porosity Variation Predicted by Various Models**

| | | Relative Estimation Error (%) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Source of Exp. Data | | Martin (1978) | | Cohen et al. (1981) | | Küfner et al. (1990) | | Mueller (1992) | | Eq. 3 | |
| | $D/d$ | $x$ | $s$ | $x$ | $s$ | $x$ | $s$ | $x$ | $s$ | $x$ | $s$ |
| Roblee et al. (1958) | 13.7 | 13 | 10 | 20 | 17 | 21 | 28 | 14 | 9 | 12 | 10 |
| | 13.7 | 13 | 11 | 22 | 18 | 21 | 31 | 11 | 10 | 11 | 9 |
| Benenati and Brosilow (1962) | 2.61 | 19 | 15 | 25 | 17 | 32 | 33 | 19 | 13 | 11 | 10 |
| | 5.6 | 12 | 6 | 19 | 10 | 20 | 27 | 8 | 7 | 6 | 5 |
| | 14.1 | 10 | 8 | 17 | 12 | 15 | 22 | 11 | 9 | 6 | 5 |
| | 20.3 | 4 | 5 | 10 | 7 | 21 | 23 | 14 | 12 | 12 | 12 |
| Ridgway and Tarbuck (1966) | 7.6 | 11 | 8 | 15 | 10 | 28 | 26 | 15 | 12 | 13 | 10 |
| | 12.7 | 8 | 6 | 8 | 7 | 21 | 21 | 14 | 10 | 14 | 12 |
| | 15.2 | 10 | 6 | 9 | 8 | 19 | 22 | 20 | 16 | 14 | 12 |
| Goodling et al. (1983) | 7.35 | 14 | 8 | 20 | 11 | 26 | 26 | 11 | 8 | 9 | 7 |
| | 8.56 | 16 | 12 | 24 | 17 | 24 | 30 | 14 | 9 | 11 | 9 |
| | 10.7 | 14 | 11 | 21 | 15 | 22 | 23 | 11 | 10 | 10 | 9 |
| Küfner and Hofmann (1990) | 4.44 | 23 | 17 | 28 | 19 | 26 | 23 | 21 | 22 | 21 | 12 |
| Mueller (1992) | 2.02 | 20 | 11 | 31 | 16 | 45 | 36 | 10 | 6 | 20 | 9 |
| | 3.96 | 20 | 13 | 27 | 15 | 29 | 31 | 13 | 10 | 8 | 11 |
| | 5.96 | 17 | 11 | 24 | 14 | 26 | 29 | 9 | 8 | 8 | 9 |
| | 7.99 | 13 | 8 | 19 | 10 | 21 | 22 | 11 | 10 | 9 | 7 |
| Average | | 14 | 10 | 20 | 13 | 25 | 27 | 13 | 11 | 11 | 9 |

ences into account and, despite the added complexity, these two regions are modeled differently (see Eqs. 3a and 3b).

$$\epsilon(r) = 2.14z^2 - 2.53z + 1,\ z \leq 0.637 \quad (3a)$$

$$\epsilon(r) = \epsilon_b + 0.29\exp(-0.6z)\cdot[\cos(2.3\pi(z-0.16))] + 0.15\exp(-0.9z),\ z > 0.637 \quad (3b)$$

The constants in the model were determined by regression analysis of literature data and empirical adjustment to yield sensible average bed porosity results (see Eq. 2). The performance of this model was compared to models from literature in terms of its ability to predict radial porosity variation (see Table 4), as well as average bed porosity (see Table 5).

The present model (Eq. 3) showed an improvement over existing models for the prediction of both radial porosity variation and average bed porosity. The sample standard deviation of the estimation errors was also smaller. Furthermore, it is interesting to note that the radial porosity model is comparable to the average bed porosity model (Eq. 1) in its ability to predict the average bed porosity. This implies that there is little advantage in using the average bed porosity model apart from computational simplicity in calculations requiring only the average bed porosity.

However, despite the improvements noted, radial porosity variation is a complex phenomenon and there are limitations in the descriptive ability of the present model:

(a) It is only valid for equal sized spheres.

(b) It has been developed for column to particle diameter ratios larger than 2.

(c) The modeling decision to use a sinusoidal description for the oscillations leads to an inadequate description of the repetitive parabola shaped oscillations observed by some researchers. This also leads to a bias towards higher bed porosity.

(d) The ability of packed beds to have more than one stable packing configuration for the same packing mode has already been noted. These differences observed as "anomalous" average bed porosity values, have different underlying radial porosity profiles. These differences, which have been observed as changes in the period and shape of the voidage oscillations, are not modeled.

(e) The numerical accuracy of the radial porosity values are dependent on the choice of the infinite diameter average bed voidage ($\epsilon_b$).

## Conclusions

The existence of multiple stable packing configurations within the same packing mode has been shown. This is especially prevalent at small column to particle diameter ratios. Multiple packing configurations result in randomly packed beds with "anomalous" average bed voidage and correspondingly different radial porosity distributions. These phenomena are not amenable to modeling and have an effect on the predictive ability of bed porosity models.

Despite the aforementioned, radial bed porosity models reported in literature were found lacking in their ability to describe the average bed porosity. It was also found that these models failed to model the changing mean of the radial voidage oscillations and the deviations from purely sinusoidal behavior. A radial porosity model was developed to redress some of these shortcomings and improved radial bed voidage $\epsilon(r)$ and average bed voidage $\epsilon$ prediction was demonstrated. It was also shown that there is no need for a separate average bed voidage model, apart from its computational simplicity in calculations requiring only the average bed porosity.

Despite the improved performance of the suggested radial bed porosity model over those reported in literature, it can be further improved by incorporating deviations from the sinusoidal form of the radial bed voidage variation.


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## Notation

$d$ = particle diameter
$D$ = column diameter
$n$ = number of experimental values or data sets
$R$ = column radius
$r$ = radial position relative to the column center line
$s$ = sample standard deviation
$x$ = average value
$z$ = nondimensional distance from the wall, $z = (R-r)/d$
$\epsilon$ = bed porosity
$\epsilon_b$ = bed porosity in the absence of wall effects
$\epsilon(r)$ = radial bed voidage variation as a function of distance $r$ from the wall

**Table 5. Prediction of Average Bed Voidage by Various Models**

| Source of Exp. Data | Relative Estimation Error (%) | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Martin (1978) | | Cohen and Metzner (1981) | | Küfner and Hofmann (1990) | | Mueller (1992) | | Eq. 3 | |
| | $x$ | $s$ | $x$ | $s$ | $x$ | $s$ | $x$ | $s$ | $x$ | $s$ |
| Carman (1937) | 4 | 6 | 4 | 5 | 7 | 7 | 3 | 1 | 4 | 2 |
| Leva and Grummer (1947) "loose" | 11 | 8 | 12 | 8 | 16 | 10 | 7 | 3 | 5 | 4 |
| Leva and Grummer (1947) "dense" | 10 | 7 | 12 | 8 | 16 | 10 | 11 | 4 | 5 | 3 |
| Experimental data, Table 2 | 6 | 4 | 6 | 5 | 9 | 7 | 8 | 3 | 4 | 3 |
| Average | 8 | 6 | 9 | 7 | 12 | 9 | 7 | 3 | 4 | 3 |

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